Linear representations of p-adic Heisenberg groups in spaces of analytic and continuous functions.

Bertin Diarra and Tongobé Mounkoro

Abstract

Let K be a complete valued field extension of the field of p-adic numbers $\mathbb{Q}_p$. Let $\mathcal{D}$ be a closed unitary subring of the valuation ring Λ_K of K. Let $\mathcal{H}(3, \mathcal{D})$ be the 3-dimensional Heisenberg group with entries in $\mathcal{D}$. We shall give a continuous linear representation of $\mathcal{H}(3, \mathcal{D})$ in the space $K < z >$ of restricted power series with cœfficients in K (= the Tate algebra in one variable, i.e. the space of analytic functions on Λ_K), analogous to Schrödinger representation of the classical Heisenberg group. On the other hand, assuming that $\mathcal{D}$ is compact, we shall obtain by the same way a continuous linear representation of the profinite group $\mathcal{H}(3, \mathcal{D})$ in the space of continuous functions $\mathcal{C}(\mathcal{D}, K)$, another analogue of Schrödinger representation. These representations are topologically irreducible. From the first representation, one obtains position and momentum bounded operators satisfying Heisenberg commutation relation and the Weyl algebra $A_1(K)$ as subalgebra of the algebra of bounded linear operators of $K < z >$. The closure $\widehat{A}_1(K)$ of $A_1(K)$ is described.